# Announcements

One OH at 10-11 today cancelled

- Short hw3 due this Friday.

  Small clarification on problem 1c pinned on Ed

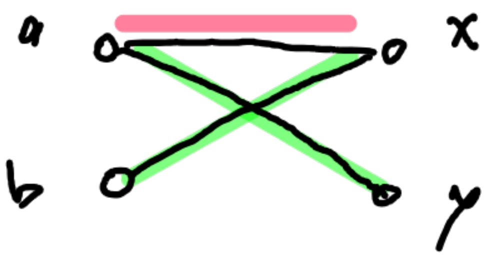
  stable matching

- Prelim 1 grades are out.

  offering 1-on-1 appointment options

- Updated course policies on the Web to include all policies (dropped quiz, regrade deadline, etc

# The maximum matching problem

x, y  a, b


preferences

stable matching (a, x)

maximum matching

today: max size matching

matching: – each node has 0 or 1 adjecent edge

Problem: Input $G = (V, E)$, $V = A \cup B$ two disjoint sides

+ $E \subseteq A \times B$ • bipartite & undirected

find $M \subseteq E$ matching

$|M|$ maximum

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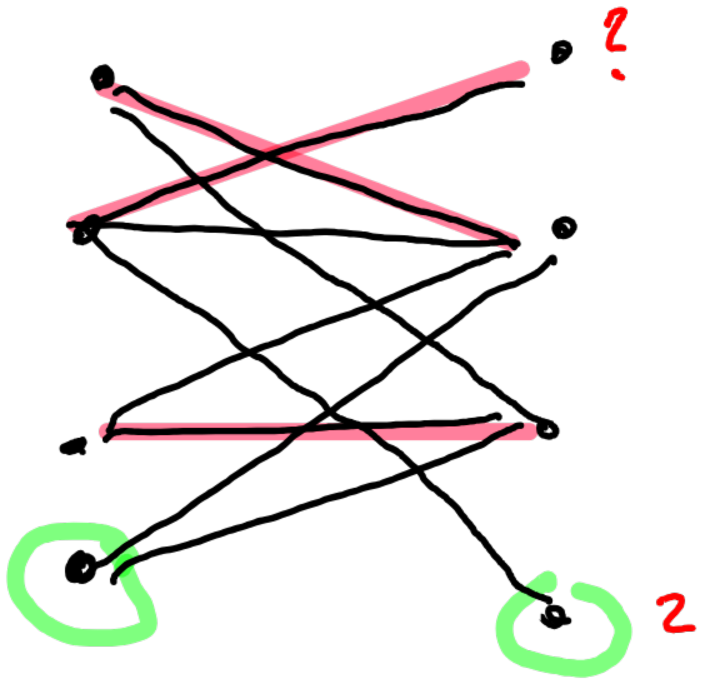
What is the size of the maximum matching in the graph given?

A. Size 1

B. Size 2

C. Size 3

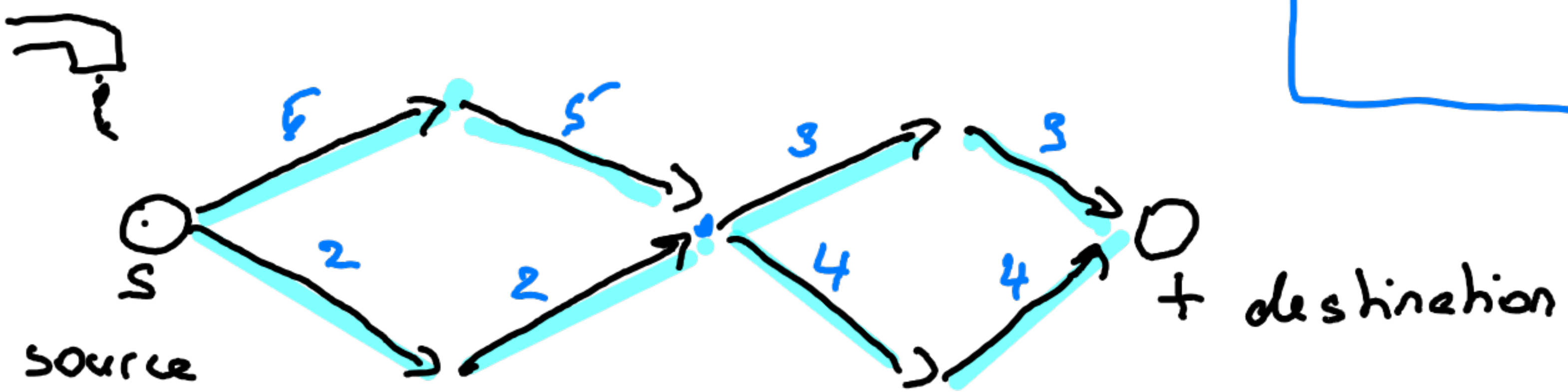
D. Size 4




max = 3

# The maximum flow problem

reduce




Input: directed $G=(V,E)$

$s, t \in V$ source & sink

$c_e \geq 0$ capacity of edge

assume s has no incoming edges

flow $f$ $\quad 0 \leq f_e \leq c_e$

amount on edge e

flow conservation:

$\forall v \neq s,t$

$$\sum_{w:(w,v)\in E} f_{wv} = \sum_{w:(v,w)\in E} f_{vw}$$

flow in = flow out

value(f)

= outflow at s

$$= \sum_{v:(s,v)\in E} f_{sv}$$

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What is the value of the maximum flow in the flow network given

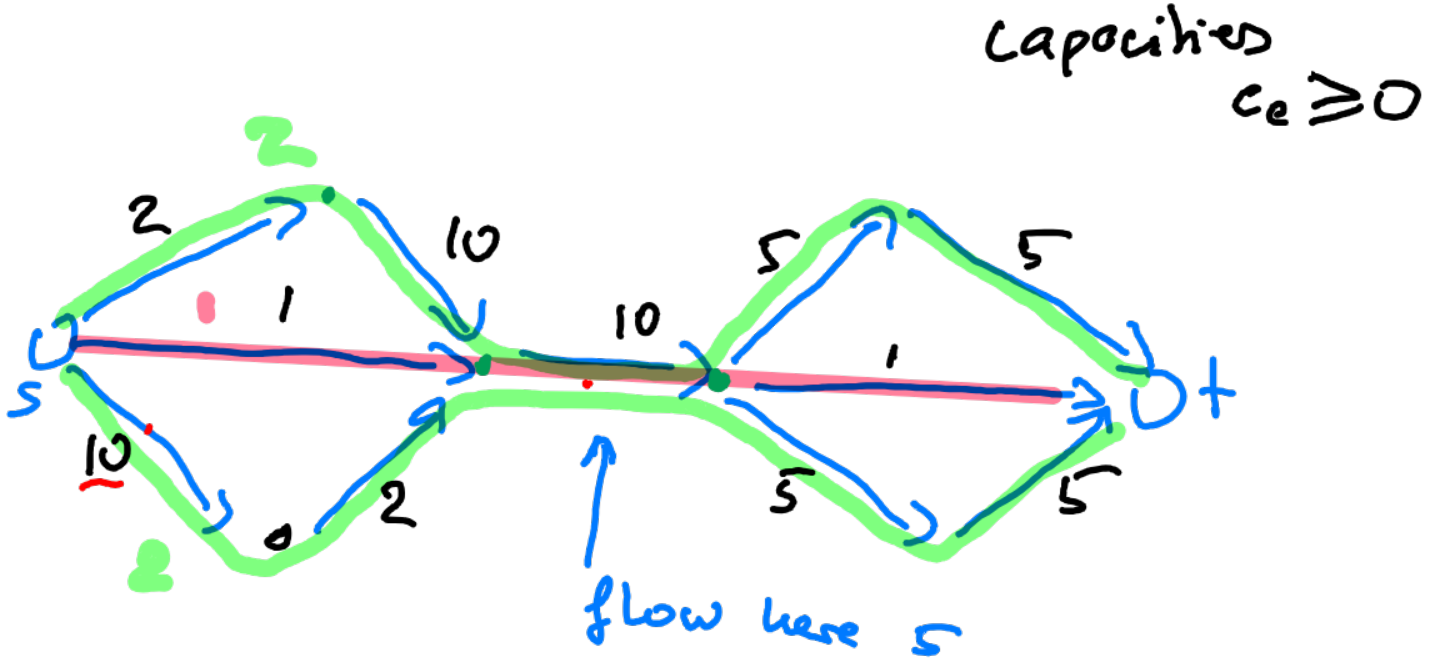
A. Max value is 1

B. Max value is 2

C. Max value is 5

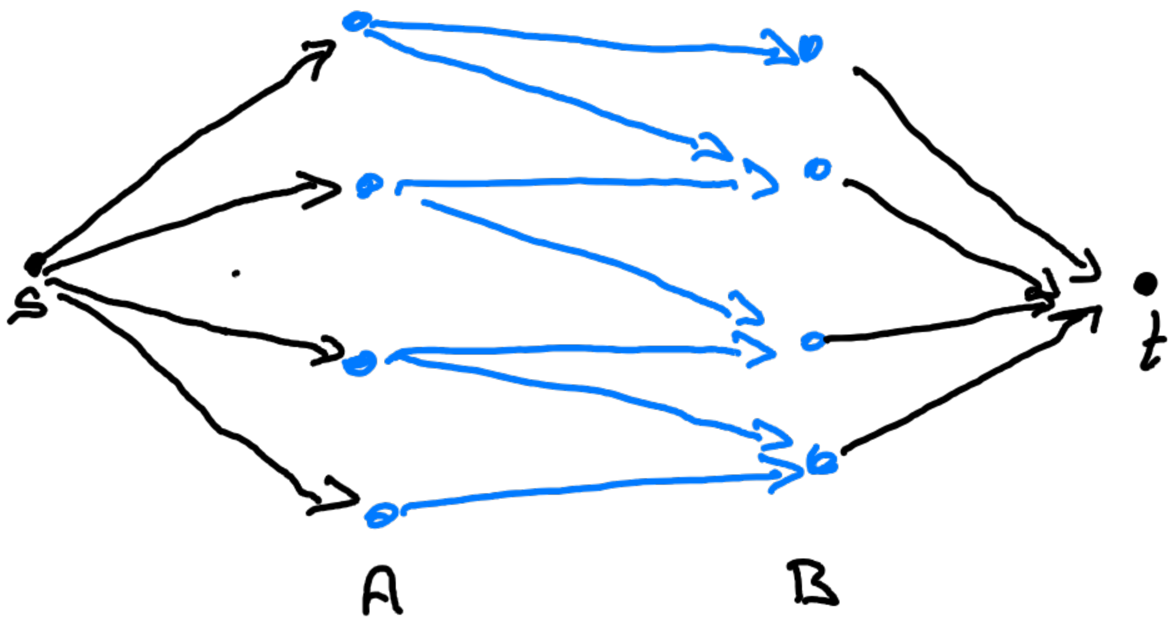
D. Max value is 8

E. Max value is 10



# Matching as maximum flow reduction

Matching input graph :




reduction:

direct all edges $A \to B$

add s + t nodes

edges $(s,v)$ for $v \in A$

$(w,t)$ for $w \in B$

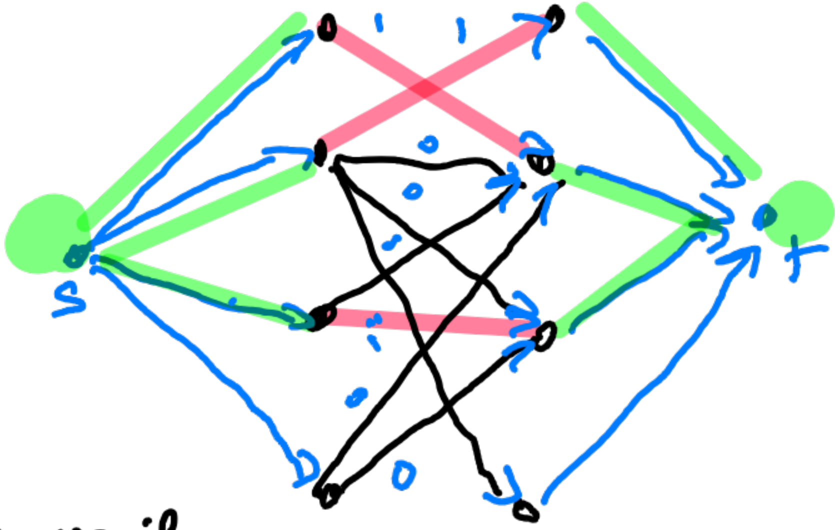
$c_e = 1$ all edges

Algorithm for max matching

1. create network as above
2. solve max flow
3. extract solution

# Proving correctness of reduction

1. $\boxed{\text{if bipartite graph has matching } M \text{ of size } m = |M| \implies \text{directed network has flow value} = m}$




then flow network has flow

$v, w \in A \times B$ in original graph

$$f_{vw} = \begin{cases} 1 & \text{if } (v,w) \in M \\ 0 & \text{otherwise} \end{cases}$$

- $$f_{sv} = \begin{cases} 1 & \text{if } v \text{ is matched in } M \\ 0 & \text{otherwise} \end{cases}$$

$$f_{wt} = \begin{cases} 1 & \text{if } w \text{ is matched in } M \\ 0 & \text{otherwise} \end{cases}$$

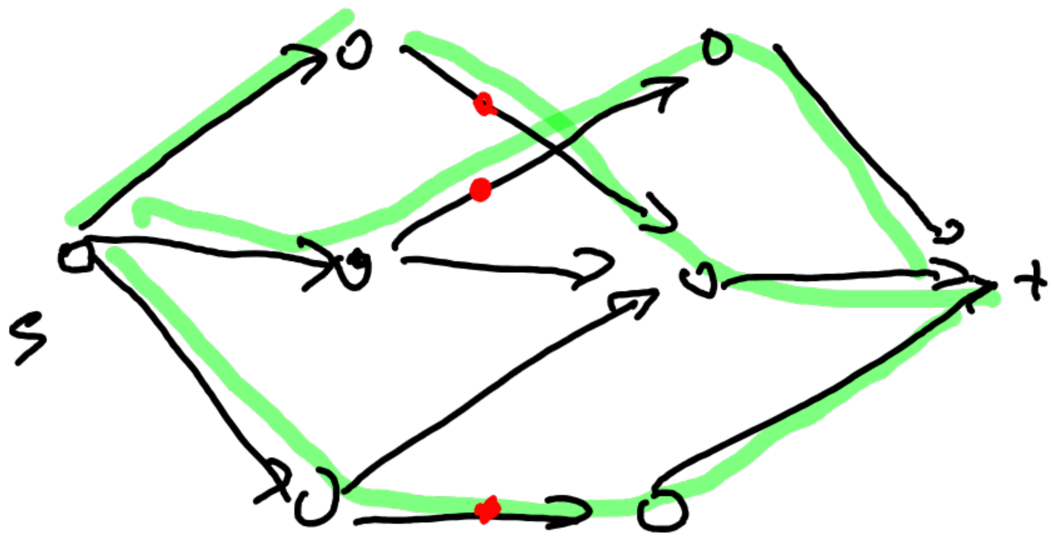
need to verify

1. flow conservation ✓
2. value of flow $= \sum_{v \in A} f_{sv} = \#$ matched nodes on A side $= |M|$

# Proving correctness of reduction (cont)

2 need: if ∃ flow of size $v$ $\Longrightarrow$ ∃ a matching of size $v$
all integer values

~~we~~ see next time:
Claim if $c_e$ integer all $e \in E$ $\Longrightarrow$ ∃ max flow with all integers




integer flow $\Rightarrow$ $f_e = \{^0_1$ as $c_e = 1$

Claim: $e \in A \times B$ such that $f_e = 1$ forms matching of size $v$

need to verify:

(1) matching, indeed due to capacity $sv$ or $wt$ edge & flow conservation

flow out of s goes to matched nodes $\overset{\text{reason}}{\Longleftrightarrow}$ (2) value $v$ = size of matching